GZ BEARING ANGLE ERROR
RESULTING FROM SENSOR SITING ERROR

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Research

~~This report develops and analyses the~~ requirements for accuracy of EM/DF site location for two bearing errors are analyzed. Results indicate that the required location accuracy is of the order of ± 100 feet.

~~It should be noted that~~ This Research ~~report~~ deals only with accuracy of antenna location and not with the accuracy of pointing.

GZ BEARING ANGLE ERROR RESULTING FROM SENSOR SITING ERROR

1.0 THE PROBLEM:

A single sensor site, S, provides, among other outputs, the bearing angle, β , of a nuclear event occurring at GZ. The angle is measured relative to a longitudinal line passing through the site in a clockwise direction as in the figure below.

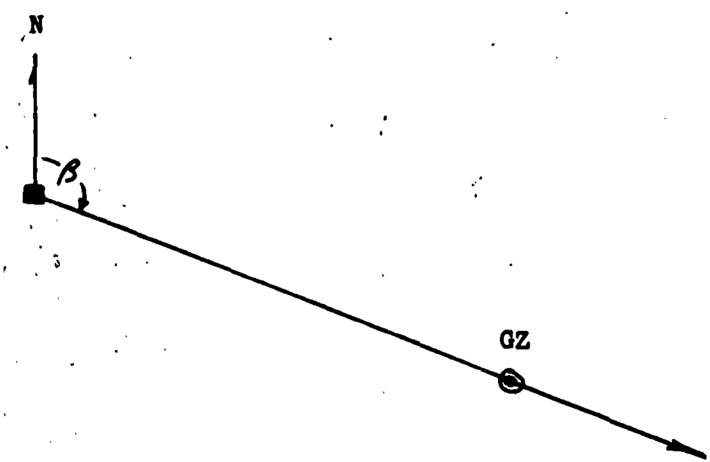

FIGURE 1 BEARING ANGLE (PLANE REPRESENTATION)

It is desired to know how accurately the position of the sensor site must be known as a function of ground range and bearing angle error. Alternatively, if the siting uncertainty and the expected distance to a nuclear detonation is known, the possible, resultant bearing angle uncertainty is desired.

2.0 RESULTS:

A simple relationship was observed to exist:

$$\tan \frac{d}{\rho} = \sin \frac{r}{\rho} \tan \Delta\beta$$

where d = Tolerable siting error

r = Ground range from site to GZ

ρ = Mean radius of earth

$\Delta\beta$ = Bearing angle error

However, for small bearing angle errors ($\Delta\beta \leq .1^{\circ}$) and for ground distances not exceeding 300 miles, the siting tolerance may be estimated within 1% using the two approximations of 1) a flat earth and 2) $\tan\theta = \theta$ for small angles. The relationship is simply

$$d = \Delta\beta\, r \qquad (\Delta\beta \text{ in radians})$$

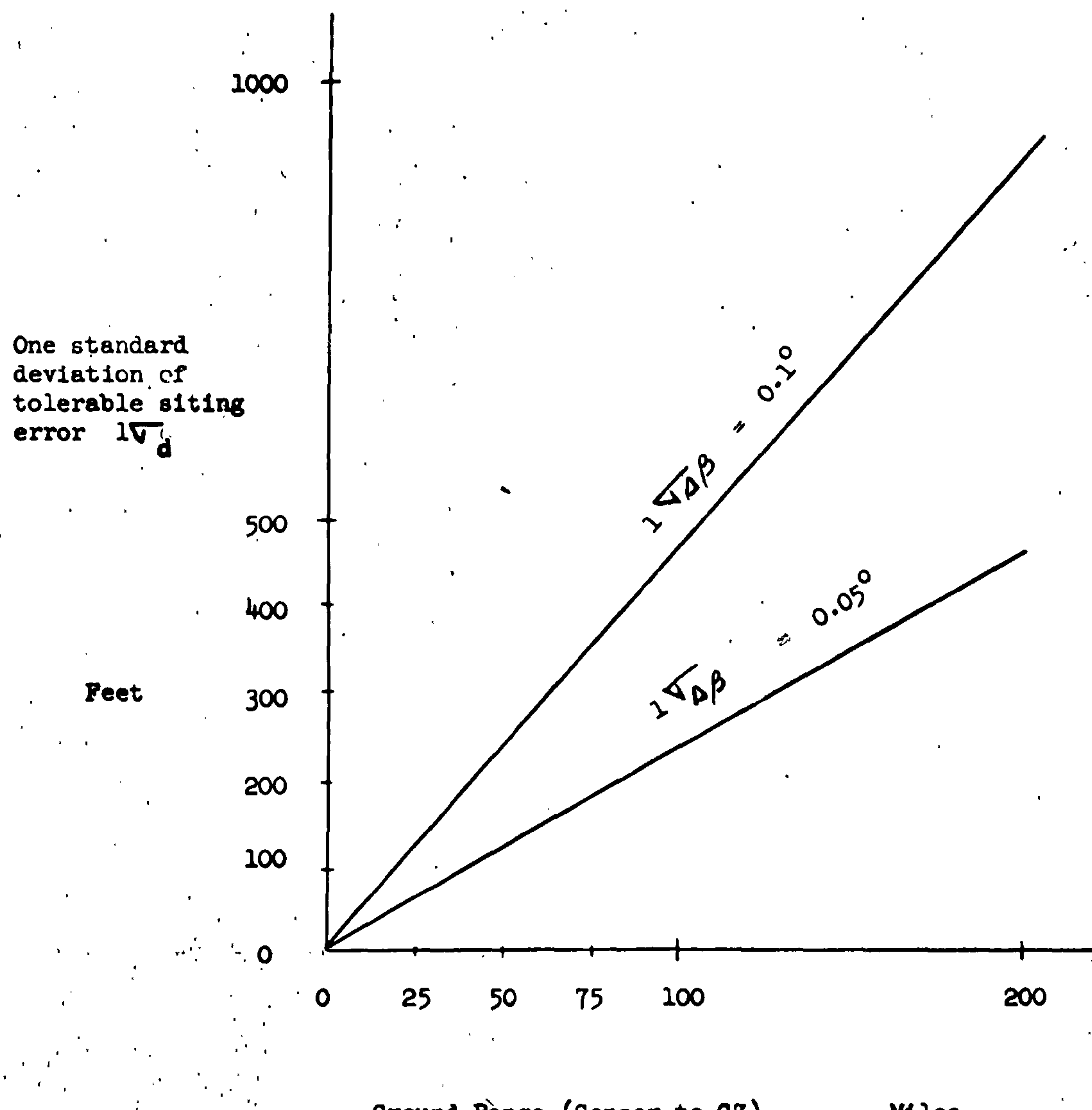

FIGURE 2 SITING ERROR VS GROUND RANGE

3.0 ANALYSIS:

3.1 The Siting Uncertainty -

It will be hypothesized that the site location is known only within certain tolerances and it may be said that the true desired location of the site lies within a bounded area.

Since the siting error may be in any direction relative to the nuclear event, further hypothesize that this error is circular with the circumference denoting one standard deviation of bearing angle error.

One standard deviation of bearing angle error is then seen to occur when the sensor is erroneously located at the point where a line passing through ground zero is tangent to the error circle (point a). However, a major assumption will be made that the 1σ will occur when the sensor site is erroneously located at the point b. Point b is where a line passing through the site, S, and perpendicular to the sensor site - ground zero line intersects the error circle. This is not really a bad assumption because, for ground distances not less than 25 miles and error circles having radii on the order of feet, points a and b are virtually identical.

The problem is now reduced to an exercise in spherical trigonometry.

3.2 Governing Relationship -

To identify the nomenclature, we may suppose that the sensor site is located at ρ_s, θ_s, ϕ_s and GZ is at ρ_g, θ_g, ϕ_g. Of course, assuming a

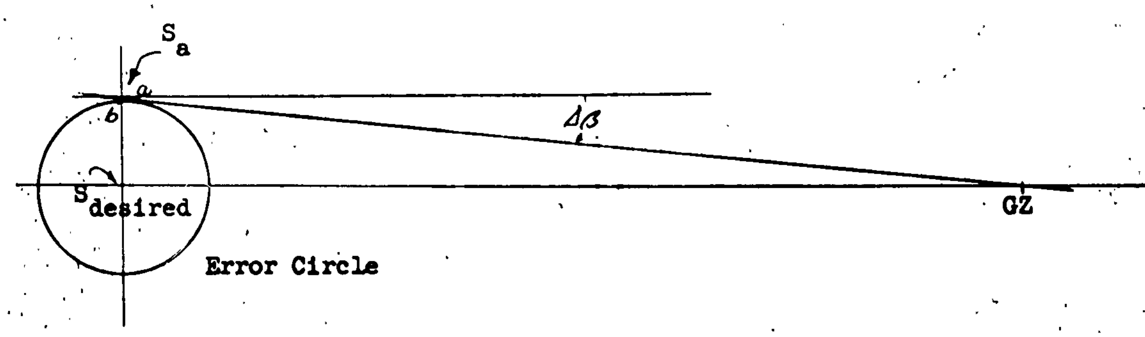

FIGURE 3 ERRONEOUS PLACEMENT OF SITE (PLANE REPRESENTATION)

spherical earth

$$\rho_s = \rho_g = \rho$$

For convenience and without degeneration of generality let

$$\theta_s = 0$$

$$\phi_g = 0$$

the pictorial representation is given in Figure 4.

If the sensor site were where desired, a bearing angle of $\beta_o = 90^\circ$ would be indicated. Since the site is actually at S_{actual} the indicated bearing angle is β_a which contains an error $\Delta\beta$.

$$\beta_a = \beta_o + \Delta\beta$$

The problem becomes trivial if it is granted that the angle denoted by α is also $\Delta\beta$ and that it is noted that the triangle formed by $S_{desired}$ - S_{actual} - GZ is a right spherical triangle. For a right spherical triangle the relationship exists

$$\tan \alpha = \frac{\tan \phi_s}{\sin \theta_g}$$

or

$$\tan \phi_s = \sin \theta_g \tan \Delta\beta_s$$

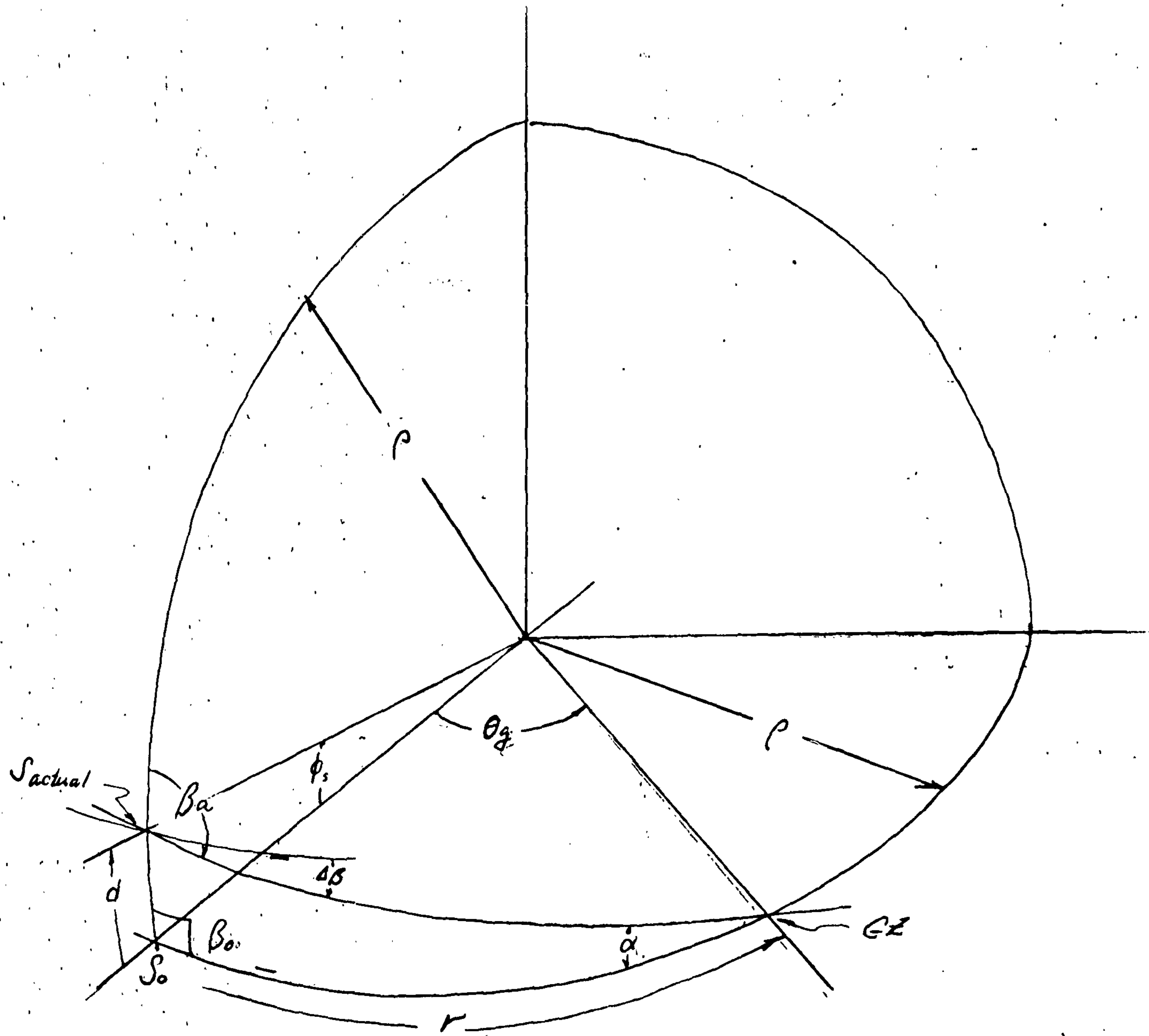

FIGURE 4 COORDINATE SYSTEM REPRESENTATION

In terms of distances, the earth central angles ϕ_s and θ_d may be written

$$\phi_s = \frac{d}{\rho}$$

$$\theta_d = \frac{r}{\rho}$$

where d = Siting error or tolerance

r = Ground range

ρ = Radius of earth

Then the governing trigonometric relationship becomes

$$\boxed{\tan\frac{d}{\rho} = \sin\frac{r}{\rho} \tan\Delta\beta_s}$$

4.0 <u>NUMERICAL DATA</u>:

Radius of Earth, ρ

3959 statute miles

2.9092×10^7 feet

Bearing Angle Error, $\Delta\beta$

$.05^\circ = 0.000873$ radians

$.10^\circ = 0.001745$ radians

Ground Range, r	
25 mi.	0.0063147 radians
50	0.0126294
100	0.0252590
200	0.0505178

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REVISION A TO

DEFLECTIONS ANALYSIS

OF

D/F ANTENNA STRUCTURE

dated

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63-2-2-2 [illegible]

Revision A
28 January 1963

TO:

See Distribution

The enclosed revision to Appendix B (pages 36, 37, 37b and 38) shall replace the previous appendix in its entirety.

The following change should be made to page 4 paragraph 4.2:

	Peak Overpressure (psi)	Peak Dynamic Pressure (psi)
	72	80
	50	40
(changed from 80)	30	16
(changed from 70)	20	8
	10	2
	5	0.7
	2	0.1

APPENDIX B

WIND PRESSURE IN TERMS OF WIND VELOCITY

An expression for fluid pressure is given by[1]

$$p = \frac{C\rho V^2}{2} \qquad \text{B.1}$$

Where:

p = Pressure (psI)

C = Dimensionless Coefficient

ρ = Air Density

V = Wind Velocity

The coefficient is a function of the Reynolds number which is an expression involving air density, velocity and viscosity. As a first approximation, let

$$C^2 = 0.5, 1, 2$$

be constant.

Also, a constant value of air density will be assumed. "Moist Air" at 25°C and 760 MM of mercury pressure has a density of [3]

$$\rho = 1.1845 \ ^{\text{Grams}}/_{\text{Liter}} \qquad \text{B.2}$$

1. Prandtl, Applied Hydro and Aero Dynamics
2. Norris, Structural Design for Dynamic Loads, p. 439

In more convenient units this is

$$\rho = 2.377 \times 10^{-3} \quad \# - SEC^2/FT^4 \qquad B.3$$

Substitution of this value of air density in expression B.1 and making suitable dimensional corrections yields

$$p = 1.776 \times 10^{-5} \; C \; V^2 \qquad B.4$$

p = Pressure $(\#/in.^2)$

v = Wind Velocity (MPH)

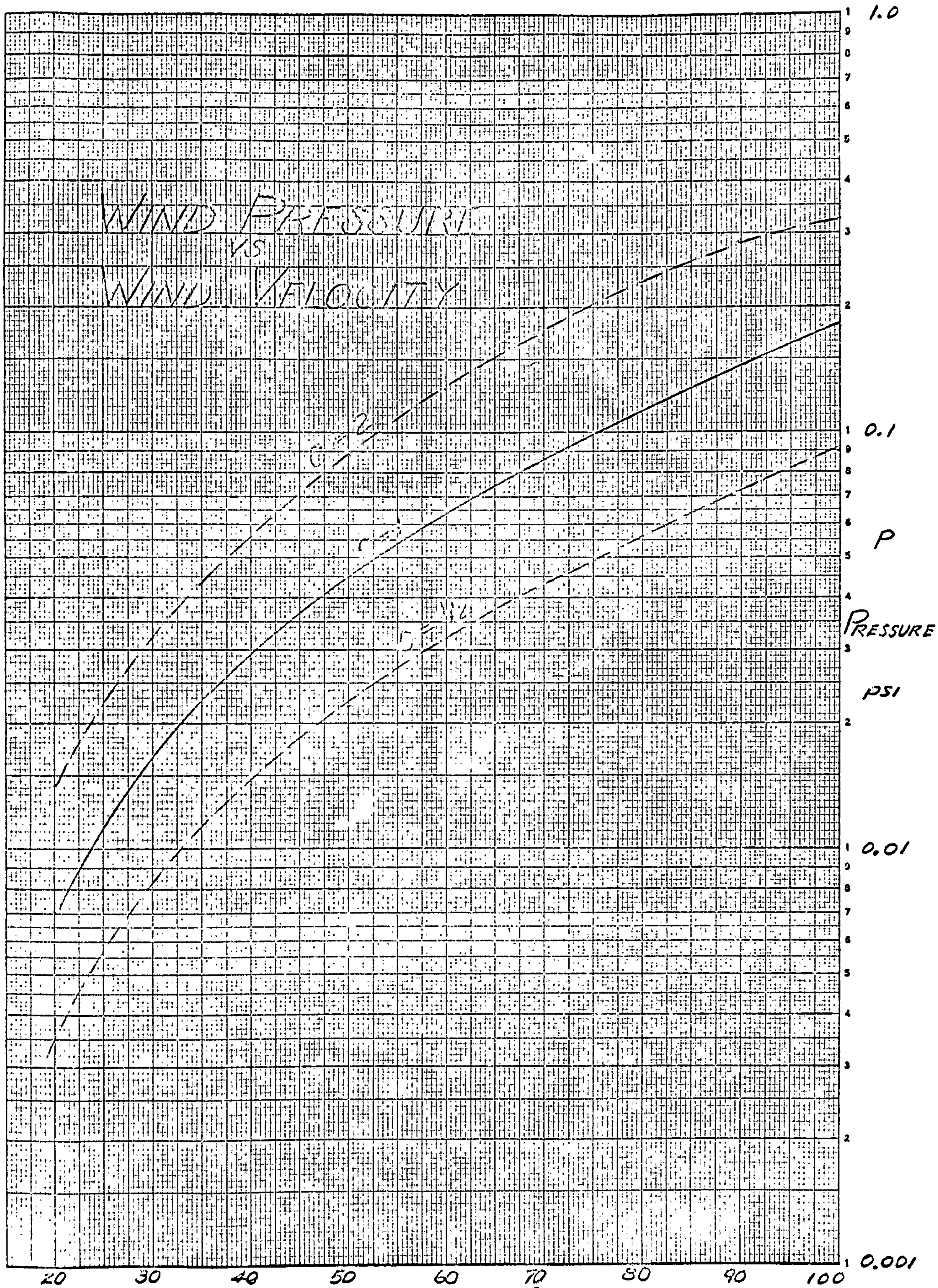
Values of pressure are tabulated in Table B.1 and presented graphically in Graph B.1

When precise data on atmospheric conditions is lacking, it is conventional to use for the drag coefficient, C, the value of unity (C = 1) for sea level and low altitude calculations.

The wind pressure vs wind velocity curve for C = 1 compares favorably to corresponding calculations in the available literature.

V (MPH)	p (psI)		
	C = .5	C = 1	C = 2
10	.0009	.0018	.0035
20	.0035	.0071	.0142
30	.0080	.0160	.0320
40	.0142	.0284	.0568
50	.0221	.0444	.0888
60	.0320	.0639	.1280
70	.0435	.0869	.1739
80	.0568	.1137	.2267
90	.0720	.1440	.2888
100	.0888	.1776	.3230

TABLE B.1 AIR PRESSURE vs WIND VELOCITY


WIND PRESSURE
VS
WIND VELOCITY
1.0
0.1
0.01
0.001
P
PRESSURE
psi
20
30
40
50
60
70
80
90
100
V WIND VELOCITY MILES PER HOUR